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# Eleven-dimensional supergravity in light-cone superspace

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Abstract: We show that Supergravity in eleven dimensions can be described in terms of a constrained superfield on the light-cone, *without the use of auxiliary fields*. We build its action to first order in the gravitational coupling constant $\kappa$, by "oxidizing" $(\mathcal{N}=8\,, d=4)$ Supergravity. This is simply achieved, as for $\mathcal{N}=4$ Yang-Mills, by extending the transverse derivatives into superspace. The eleven-dimensional SuperPoincaré algebra is constructed and a fourth order interaction is conjectured.

Keywords: Field Theories in Higher Dimensions, Superspaces.

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## 1. Introduction

$\mathcal{N}=1$ supergravity in eleven dimensions [1], is the largest supersymmetric local field theory with maximum helicity two (on reduction to $d=4$). This theory has gained renewed prominence since its recognition as the infrared limit of M-theory [2]. Although M-theory casts well-defined shadows on lower-dimensional manifolds, its actual structure remains a mystery. We must therefore glean all we can from the $\mathcal{N}=1$ supergravity theory before tackling M-theory. $\mathcal{N}=1$ supergravity is ultraviolet divergent in $d=11$ but this divergence is presumably tamed by M-theory and the hope is that an understanding of this divergent structure, will give us a window into the workings of M-theory.

Since M-theory resides in $d=11$, we expect special features of eleven-dimensional spacetime to be reflected in its physical little group, SO(9). Curtright [3] conjectured that the divergences that occur in ($\mathcal{N}=8$, $d=4$) supergravity (obtained by reducing $d=11$ supergravity) were a direct consequence of the group-theoretical properties of SO(9). Indeed, the mismatch between the eight-order bosonic and fermionic Dynkin indices seems to support this conjecture. The best approach to divergence-analysis, keeping the role of the little group apparent, is on the light-cone. We therefore need to formulate eleven-dimensional supergravity in light-cone gauge before starting an analysis of its divergences. Gravity in light-cone gauge has been studied previously by numerous authors (a partial list includes, [4−7]).

This paper represents the second step in our research program, initiated in reference [8], where we described the oxidation of ($\mathcal{N}=4, d=4$) SuperYang-Mills into ($\mathcal{N}=1, d=10$) Yang-Mills. In this paper, we start by building the eleven-dimensional SuperPoincaré generators. We then generalize the notion of the transverse derivative into superspace and show that this simple generalization, uniquely oxidizes ($\mathcal{N}=8, d=4$) Supergravity into the fully eleven-dimensional $\mathcal{N}=1$ Supergravity (up to first order in the coupling constant).

In a recent paper Metsaev [9] discussed the same theory in a somewhat different light-cone formulation, which was originally used by Brink, Green and Schwarz [10] in their formulation of a light-cone gauge field theory for the $d=10$ superstring. We believe that the two approaches are valuable complements in the study of $d=11$ supergravity.

## 2. The $LC_2$ formulation of $\mathcal{N}=8$

Our starting point for this paper is the 1982 light-cone formulation initiated in reference [11]. Based on the fact that both the ($\mathcal{N}=8$, $d=4$) Supergravity theory and the ($\mathcal{N}=4$, $d=4$) SuperYang-Mills theory are maximally supersymmetric, these authors introduced a constrained superfield for the $\mathcal{N}=8$ theory. We start with this superfield and show that it may be used to describe the ($\mathcal{N}=1\,, d=11$) Supergravity theory in light-cone superspace.

### 2.1 Field content

$\mathcal{N}=1$ Supergravity in eleven dimensions, contains three different massless fields, two bosonic (gravity and a three-form) and one Rarita-Schwinger spinor. Its physical degrees of freedom are classified in terms of the transverse little group, SO(9), with the Graviton $G^{(MN)}$, transforming as a symmetric second-rank tensor, the three-form $B^{[MNP]}$ as an anti-symmetric third-rank tensor and the Rarita-Schwinger field as a spinor-vector, $\Psi^M$($M, N, \ldots$ are SO(9) indices). This theory on reduction to four dimensions leads to the maximally supersymmetric $\mathcal{N}=8$ theory.

In $d=4$, any massless particle can be described by a complex field, and its complex conjugate of opposite helicity under the SO(2) little group. In this case, the SO(2) comes from the decomposition,

$$\mathrm{SO}(9) \supset \mathrm{SO}(2) \times \mathrm{SO}(7)\,. \tag{2.1}$$

All told, after decomposition, the $\mathcal{N}=8$ theory has a spectrum comprised of a metric, twenty-eight vector fields, seventy scalar fields, fifty-six spin one-half fields, eight spin three-half fields and their conjugates [12]. The SO(7) symmetry is an internal one and can in fact be upgraded to an SU(8) symmetry. However, it is important to remember that it is really the SO(7) which is relevant when we "oxidize" the theory to $d=11$.

*All* the physical degrees of freedom of the $\mathcal{N}=8$ theory are captured by a single complex superfield [11],

$$\begin{aligned}\phi\,(y) \;=\; & \frac{1}{\partial^{+2}}\,h\,(y)+i\,\theta^{\alpha}\,\frac{1}{\partial^{+2}}\,\bar{\psi}_{\alpha}\,(y)+i\,\theta^{\alpha\,\beta}\,\frac{1}{\partial^{+}}\,\bar{A}_{\alpha\,\beta}\,(y)\,- \\ & -\,\theta^{\alpha\,\beta\,\gamma}\,\frac{1}{\partial^{+}}\,\bar{\chi}_{\alpha\,\beta\,\gamma}\,(y)-\,\theta^{\alpha\,\beta\,\gamma\,\delta}\,C_{\alpha\,\beta\,\gamma\,\delta}\,(y)+i\,\tilde{\theta}^{(5)}_{\alpha\,\beta\,\gamma}\,\chi^{\alpha\,\beta\,\gamma}(y)\,+ \\ & +\,i\,\tilde{\theta}^{(6)}_{\alpha\,\beta}\,\partial^{+}\,A^{\alpha\,\beta}(y)+\,\tilde{\theta}^{(7)}_{\alpha}\,\partial^{+}\,\chi^{\alpha}(y)+\tilde{\theta}^{(8)}\,\partial^{+2}\,\bar{h}\,(y)\,, \end{aligned} \tag{2.2}$$

where,

$$\theta^{\alpha_1\,\cdots\,\alpha_n}\;\equiv\;\frac{1}{n!}\,\theta^{\alpha_1}\,\cdots\,\theta^{\alpha_n}\qquad \tilde{\theta}^{(n)}_{\beta_1\,\cdots\,\beta_{8-n}}\;\equiv\;\frac{1}{n!}\,\theta^{\alpha_1\,\cdots\,\alpha_n}\,\epsilon_{\alpha_1\,\cdots\,\alpha_n\,\beta_1\,\cdots\,\beta_{8-n}}\,. \tag{2.3}$$

The $\gamma$-matrices allow us to translate bi-spinor indices into vector indices and vice versa (For our notation and light-cone conventions, we refer the reader to the Appendix). For example, the complex $\bar{A}_{\alpha\,\beta}$ represent,

$$\theta^{\alpha\,\beta}\,\bar{A}_{\alpha\,\beta}=-\,\frac{1}{8}\,\theta\,\gamma^{m}\,\theta\,\bar{A}_{m}-\,\frac{1}{8}\,\theta\,\gamma^{mn}\,\theta\,\bar{A}_{[mn]}\,, \tag{2.4}$$

while the real $C_{\alpha\,\beta\,\gamma\,\delta}$ may be decomposed as,

$$\theta^{\alpha\,\beta\,\gamma\,\delta}\,C_{\alpha\,\beta\,\gamma\,\delta}=\frac{1}{64}\,\theta\,\gamma^{m}\,\theta\,\left\{\;\theta\,\gamma^{p}\,\theta\,C_{(mp)}\,-\,\theta\,\gamma^{mp}\,\theta\,C_{p}\;-\,\theta\,\gamma^{pq}\,\theta\,C_{[mpq]}\right\}. \tag{2.5}$$

These tensor fields, make up the two bosonic representations under the decomposition $\mathrm{SO}(9)\subset\mathrm{SO}(7)\,\times\,\mathrm{SO}(2)$ of eleven-dimensional supergravity. The $G^{(MN)}$ (**44** of SO(9)), split up as,

$$\mathbf{44}\;=\;\mathbf{27}+\mathbf{7}+\mathbf{7}+\mathbf{1}+\mathbf{1}+\mathbf{1}\,. \tag{2.6}$$

$C_{(mp)}$ represents the **27** while $\bar{A}_m$, $A^m$ represent the **7** + **7**. Similarly, the three-form $B^{[MNP]}$ (**84** of SO(9)) splits into,

$$\mathbf{84}\;=\;\mathbf{35}+\mathbf{21}+\mathbf{21}+\mathbf{7}\,. \tag{2.7}$$

These correspond to $C_{[mpq]}$, $\bar{A}_{[mn]}$, $A_{[mn]}$ and $C_p$ respectively. All fields are local in the modified light-cone coordinates

$$y=\left(\,x,\,\bar{x},\,x^{+},\,y^{-}\equiv x^{-}\,-\,\frac{i}{\sqrt{2}}\,\theta^{\alpha}\,\bar{\theta}_{\alpha}\,\right). \tag{2.8}$$

In this $LC_2$ form, all the unphysical degrees of freedom have been integrated out. The superfield $\phi$ and its complex conjugate $\bar{\phi}$ satisfy the chiral constraints,

$$d^{\alpha}\phi=0\,;\qquad \bar{d}_{\alpha}\,\bar{\phi}=0\,, \tag{2.9}$$

and are related through the "inside-out" constraints,

$$\phi=\frac{1}{8!\,\cdot\,4}\,(d)^{8}\,\frac{1}{\partial^{+4}}\bar{\phi}\,. \tag{2.10}$$

The $\mathcal{N}=8$ Supergravity action, to order $\kappa$ is then simply [13],

$$\int d^4x \int d^8\theta\, d^8\bar{\theta}\, \mathcal{L} \;\equiv\; \int \mathcal{L}\,, \tag{2.11}$$

where,

$$\mathcal{L} = -\bar{\phi}\,\frac{\Box}{\partial^{+4}}\,\phi + \left(\frac{4\,\kappa}{3\,\partial^{+4}}\,\overline{\phi}\,\bar{\partial}\bar{\partial}\,\phi\,\partial^{+2}\phi - \frac{4\,\kappa}{3\,\partial^{+4}}\,\overline{\phi}\,\bar{\partial}\,\partial^{+}\phi\,\bar{\partial}\,\partial^{+}\,\phi + \text{c.c.}\right). \tag{2.12}$$

Grassmann integration is normalized so that $\int d^8\theta\,(\theta)^8 = 1$.

**A simpler $\mathcal{N}=8$ Three-point vertex**

The three-point vertex in this action, seems highly non-local and cumbersome. However, its form can be greatly simplified, leading to a single term, very similar to that in the Yang-Mills case [8]. We start by partially integrating the first term with respect to $\bar{\partial}$, to obtain

$$\int \left\{-\frac{\bar{\partial}}{\partial^{+4}}\,\overline{\phi}\;\;\bar{\partial}\,\phi\,\partial^{+2}\,\phi - \frac{1}{\partial^{+4}}\,\overline{\phi}\;\;\bar{\partial}\,\phi\bar{\partial}\partial^{+2}\,\phi\right\}\,. \tag{2.13}$$

The last $\phi$ in the first term of equation (2.13), may be rewritten as a $\overline{\phi}$, using the inside-out relation,

$$\phi = \frac{1}{8!\,\cdot 4}\,(d)^8\,\frac{1}{\partial^{+4}}\bar{\phi}\,. \tag{2.14}$$

We then partially integrate the $(d)^8$ onto the $\overline{\phi}$ and use the inside-out relation again. The second term in equation (2.13) is partially integrated with respect to $\partial^+$ to yield two terms. Thus the first term in the 3-point vertex is now,

$$\int \left\{-\bar{\partial}\,\phi\;\;\bar{\partial}\,\phi\,\frac{1}{\partial^{+2}}\,\overline{\phi} + \frac{1}{\partial^{+3}}\,\overline{\phi}\;\;\bar{\partial}\,\phi\,\bar{\partial}\,\partial^{+}\,\phi + \frac{1}{\partial^{+4}}\,\overline{\phi}\;\;\bar{\partial}\,\partial^{+}\,\phi\,\bar{\partial}\,\partial^{+}\,\phi\right\}\,. \tag{2.15}$$

The third term in this equation exactly cancels the second term in the original vertex. Next, we eliminate the middle term in the equation above, by recognizing that,

$$I = \int \frac{1}{\partial^{+3}}\,\overline{\phi}\;\;\bar{\partial}\,\phi\,\bar{\partial}\,\partial^{+}\,\phi \;=\; -\int \frac{1}{\partial^{+2}}\,\overline{\phi}\;\;\bar{\partial}\,\phi\,\bar{\partial}\,\phi - I\,, \tag{2.16}$$

(which follows from a partial integration of the single $\partial^+$ in the numerator). This allows us to set,

$$I = -\int \frac{1}{2}\,\frac{1}{\partial^{+2}}\,\overline{\phi}\;\;\bar{\partial}\,\phi\,\bar{\partial}\,\phi\,. \tag{2.17}$$

We thus obtain, a very concise three-point vertex,

$$-2\kappa\int \frac{1}{\partial^{+2}}\,\overline{\phi}\;\;\bar{\partial}\,\phi\,\bar{\partial}\,\phi + \text{c.c.}\,. \tag{2.18}$$

This simple form allows for comparison with the $\mathcal{N}=4$ SuperYang-Mills three-point vertex [11],

$$\frac{4}{3}g\int f^{abc}\,\frac{1}{\partial^{+}}\,\overline{\phi}^{a}\;\phi^{b}\,\bar{\partial}\,\phi^{c} + \text{c.c.}\,. \tag{2.19}$$

We will exploit this similarity in structure, between the $\mathcal{N}=8$ and $\mathcal{N}=4$ cases, when tackling the four-point vertex.



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### 2.2 SuperPoincaré algebra

As usual we will consider the generators of the SuperPoincaré algebra at the light-cone time $x^+ = 0$. The SuperPoincaré algebra, splits up into kinematical and dynamical pieces. The kinematical generators are,

- the three momenta,

$$p^{+}_{=} - i\,\partial^+\,, \qquad p = -i\,\partial\,, \qquad \bar{p} = -i\,\bar{\partial}\,, \tag{2.20}$$

- the transverse space rotation,

$$j = x\,\bar{\partial} - \bar{x}\,\partial + S^{12}\,, \tag{2.21}$$

where,

$$S^{12} = \frac{1}{2}\,(\,\theta^\alpha\,\bar{\partial}_\alpha - \bar{\theta}_\alpha\,\partial^\alpha\,) - \frac{i}{4\sqrt{2}\,\partial^+}\,(\,d^\alpha\,\bar{d}_\alpha - \bar{d}_\alpha\,d^\alpha\,)\,. \tag{2.22}$$

This form is different from that in reference [13] through the last term, which acts as a helicity counter. It also ensures that the chirality constraints are preserved,

$$\left[j\,, d^\alpha\right] = \left[j\,, \bar{d}_\beta\right] = 0\,. \tag{2.23}$$

- and the "plus-rotations",

$$j^+ = i\,x\,\partial^+\,, \qquad \bar{j}^+ = i\,\bar{x}\,\partial^+\,. \tag{2.24}$$

$$j^{+-} = i\,x^-\,\partial^+ - \frac{i}{2}\left(\theta^\alpha\bar{\partial}_\alpha + \bar{\theta}_\alpha\,\partial^\alpha\right)\,, \tag{2.25}$$

which satisfy,

$$\begin{aligned} \left[\,j^{+-}, y^-\right] &= -i\,y^-\,, \\ \left[j^{+-}\,, d^\alpha\right] &= \frac{i}{2}\,d^\alpha\,, \qquad \left[j^{+-}\,, \bar{d}_\beta\right] = \frac{i}{2}\,\bar{d}_\beta\,, \end{aligned} \tag{2.26}$$

and thus preserve chirality. Note that it is only for the choice $x^+ = 0$ that the generator $j^{+-}$ is kinematical, since the dynamical part is multiplied by $x^+ = 0$.

The dynamical generators are,

- the light-cone hamiltonian,

$$p^- = -i\frac{\partial\bar{\partial}}{\partial^+} \tag{2.27}$$

- and the dynamical boosts,

$$\begin{aligned} j^- &= i\,x\,\frac{\partial\bar{\partial}}{\partial^+} - i\,x^-\,\partial + i\,\left(\theta^\alpha\bar{\partial}_\alpha + \frac{i}{4\sqrt{2}\,\partial^+}\left(d^\alpha\,\bar{d}_\alpha - \bar{d}_\alpha\,d^\alpha\right)\right)\frac{\partial}{\partial^+}\,, \\ \bar{j}^- &= i\,\bar{x}\,\frac{\partial\bar{\partial}}{\partial^+} - i\,x^-\,\bar{\partial} + i\,\left(\bar{\theta}_\beta\partial^\beta + \frac{i}{4\sqrt{2}\,\partial^+}\left(d^\beta\,\bar{d}_\beta - \bar{d}_\beta\,d^\beta\right)\right)\frac{\bar{\partial}}{\partial^+}\,. \end{aligned} \tag{2.28}$$

The helicity counter added to the expressions above, once again ensures that they leave chirality and the inside-out relations unaltered because,

$$[\,j^-\,,\,d^\alpha\,]=\frac{i}{2}\,d^\alpha\,\frac{\partial}{\partial^+}\,,\qquad [\,j^-\,,\,\bar{d}_\beta\,]=\frac{i}{2}\,\bar{d}_\beta\,\frac{\partial}{\partial^+}\,. \tag{2.29}$$

These generators also satisfy,

$$[\,j^-\,,\,\bar{j}^+\,]=-i\,j^{+-}-j\,,\qquad [\,j^-\,,\,j^{+-}\,]=i\,j^-\,. \tag{2.30}$$

In a similar fashion, the supersymmetries split into,

- kinematical supersymmetries,

$$q^{\,\alpha}_{\,+}=-\partial^\alpha+\frac{i}{\sqrt{2}}\,\theta^\alpha\,\partial^+\,;\qquad \bar{q}_{+\beta}=\quad\bar{\partial}_\beta-\frac{i}{\sqrt{2}}\,\bar{\theta}_\beta\,\partial^+\,, \tag{2.31}$$

  satisfying

$$\{\,q^{\,\alpha}_{\,+}\,,\,\bar{q}_{+\beta}\,\}=i\,\sqrt{2}\,\delta^\alpha{}_\beta\,\partial^+\,, \tag{2.32}$$

  and anticommuting with the chiral derivatives

$$\{\,q^{\,\alpha}_{\,+}\,,\,\bar{d}_\beta\,\}=\{\,d^\alpha\,,\,\bar{q}_{+\beta}\,\}=0\,. \tag{2.33}$$

- dynamical supersymmetries, which may be obtained by boosting the kinematical ones

$$q^{\,\alpha}_{\,-}\equiv i\,[\,\bar{j}^-\,,\,q^{\,\alpha}_{\,+}\,]=\frac{\bar{\partial}}{\partial^+}\,q^{\,\alpha}_{\,+}\,,\qquad \bar{q}_{-\beta}\;\equiv\;i\,[\,j^-\,,\,\bar{q}_{+\beta}\,]=\frac{\partial}{\partial^+}\,\bar{q}_{+\beta}\,. \tag{2.34}$$

  They obey,

$$\left\{q^{\,\alpha}_{\,-}\,,\,\bar{q}_{-\beta}\right\}\;=i\,\sqrt{2}\,\delta^\alpha{}_\beta\,\frac{\partial\bar{\partial}}{\partial^+}\,, \tag{2.35}$$

$$\left\{q^{\,\alpha}_{\,+}\,,\,\bar{q}_{-\beta}\right\}\;=i\,\sqrt{2}\,\delta^\alpha{}_\beta\,\partial\,. \tag{2.36}$$

Except for the changes to ensure chirality, these operators all appear in reference [13]. In the next section, we generalize this four-dimensional algebra to eleven dimensions.

## 3. Eleven dimensions

The main aim of this paper is to show that the fully interacting ($\mathcal{N}=8, d=4$) theory can be restored to its eleven-dimensional progenitor, without altering the superfield. This enables us to formulate the ($\mathcal{N}=1, d=11$) theory without auxiliary fields.

The first step is to generalize the transverse variables to nine. In the Yang-Mills case, the compactified SO(6) was easily described by SU(4) parameters and we made use of the convenient bi-spinor notation. In the present case, the compactified SO(7) has no equivalent unitary group so we simply introduce additional real coordinates, $x^m$ and their derivatives $\partial^m$(where $m$ runs from 4 through 10). The chiral superfield remains unaltered, except for the added dependence on the extra coordinates

$$h(y)=h(x,\bar{x},x^m,y^-)\,,\qquad \text{etc}\;\ldots\,. \tag{3.1}$$

These extra variables will be acted on by new operators that will restore the higher-dimensional symmetries.

### 3.1 The superPoincaré algebra in 11 dimensions

The SuperPoincaré algebra needs to be generalized from its four-dimensional version. The SO(2) generators stay the same and we propose generators of the coset SO(9)/(SO(2) × SO(7)), of the form,

$$
\begin{aligned}
J^m &= -i\,(\,x\,\partial^m - x^m\,\partial\,) + \frac{i}{2\sqrt{2}}\,\partial^+\,\theta^\alpha\,(\gamma^m)_{\alpha\,\beta}\,\theta^\beta - \frac{i}{\sqrt{2}\,\partial^+}\,\partial^\alpha\,(\gamma^m)_{\alpha\,\beta}\,\partial^\beta + \\
&\quad + \frac{i}{2\sqrt{2}\,\partial^+}\,d^\alpha\,(\gamma^m)_{\alpha\,\beta}\,d^\beta \qquad (3.2)\\
\overline{J}^{\,n} &= -i\,(\,\bar{x}\,\partial^n - x^n\,\bar{\partial}\,) + \frac{i}{2\sqrt{2}}\,\partial^+\,\bar{\theta}_\alpha\,(\gamma^n)^{\alpha\,\beta}\,\bar{\theta}_\beta - \frac{i}{\sqrt{2}\,\partial^+}\,\bar{\partial}_\alpha\,(\gamma^n)^{\alpha\,\beta}\,\bar{\partial}_\beta + \\
&\quad + \frac{i}{2\sqrt{2}\,\partial^+}\,\bar{d}_\alpha\,(\gamma^n)^{\alpha\,\beta}\,\bar{d}_\beta \qquad (3.3)
\end{aligned}
$$

which satisfy the SO(9) commutation relations,

$$
\begin{aligned}
[J\,,\,J^m] &= J^m\,, \qquad \left[J\,,\,\bar{J}^n\right] = -\bar{J}^n \\
[J^{pq}\,,\,J^m] &= \delta^{pm}\,J^q - \delta^{qm}\,J^p \\
\left[J^m\,,\,\bar{J}^n\right] &= i\,J^{mn} + \delta^{mn}\,J\,,
\end{aligned}
\qquad (3.4)
$$

where $J$ is the same as before, $J = j$ and the SO(7) generators read,

$$
\begin{aligned}
J^{mn} &= -i(\,x^m\,\partial^n - x^n\,\partial^n\,) + \theta^\alpha\,(\gamma^m)^{\alpha\;\beta}\,(\gamma^n)^{\beta\;\sigma}\,\bar{\partial}_\sigma + \\
&\quad + \bar{\theta}_\alpha\,(\gamma^m)^{\alpha\;\beta}\,(\gamma^n)^{\beta\;\sigma}\,\partial^\sigma - \frac{1}{\sqrt{2}\,\partial^+}\,d^\alpha\,(\gamma^m)^{\alpha\;\beta}\,(\gamma^n)^{\beta\;\sigma}\,\bar{d}_\sigma\,.
\end{aligned}
\qquad (3.5)
$$

The full SO(9) transverse algebra is generated by $J\,,\,J^{mn}\,,\,J^m$ and $\bar{J}^n$. All rotations are specially constructed to preserve chirality. For example,

$$
[J^m\,,\,\bar{d}_\alpha] = 0\;; \qquad [\bar{J}^n\,,\,d^\alpha] = 0\,. \qquad (3.6)
$$

The remaining kinematical generators do not get modified,

$$
J^+ = j^+\,, \qquad J^{+-} = j^{+-}\,, \qquad (3.7)
$$

while new kinematical generators appear,

$$
J^{+\,m} = i\,x^m\,\partial^+\;; \qquad \bar{J}^{+\,n} = i\,\bar{x}^n\,\partial^+\,. \qquad (3.8)
$$

We generalize the linear part of the dynamical boosts to,

$$
\begin{aligned}
J^- &= ix\frac{\partial\bar{\partial} + \frac{1}{2}\partial^m\,\partial^m}{\partial^+} - i\,x^-\partial + i\,\frac{\partial}{\partial^+}\left\{\theta^\alpha\,\bar{\partial}_\alpha + \frac{i}{4\sqrt{2}\,\partial^+}\left(d^\alpha\bar{d}_\alpha - \bar{d}_\alpha\,d^\alpha\right)\right\} - \\
&\quad - \frac{1}{4}\,\frac{\partial_m}{\partial^+}\left\{\partial^+\,\theta^\alpha\,(\gamma^m)_{\alpha\,\beta}\,\theta^\beta - \frac{2}{\partial^+}\,\partial^\alpha(\gamma^m)_{\alpha\,\beta}\,\partial^\beta + \frac{1}{\partial^+}d^\alpha(\gamma^m)_{\alpha\beta}d^\beta\right\}.
\end{aligned}
\qquad (3.9)
$$

The other boosts may be obtained by using the SO(9)/(SO(2) × SO(7)) rotations,

$$
J^{-\,m} = [\,J^-\,,\,J^m\,]\;; \qquad \bar{J}^{-\,n} = [\,\bar{J}^-\,,\,\bar{J}^n]\,. \qquad (3.10)
$$



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We do not show their explicit forms as they are too cumbersome. The dynamical supersymmetries are obtained by boosting

$$[\,J^-\,,\,\bar q_{+\,\eta}\,]\;\equiv\;\overline{\mathcal{Q}}_\eta\;=\;-\,i\,\frac{\partial}{\partial^+}\,\overline{q}_{+\,\eta}\;-\;\frac{i}{\sqrt{2}}\,(\,\gamma^n\,)_{\eta\,\rho}\,q_+^{\;\rho}\,\frac{\partial^n}{\partial^+}\,,$$

$$[\,\bar J^-\,,q_+^\alpha\,]\equiv\mathcal{Q}^\alpha\;=\;i\frac{\bar\partial}{\partial^+}\,q_+^{\;\alpha}\;+\;\frac{i}{\sqrt{2}}\,(\,\gamma^m\,)^{\alpha\,\beta}\,\bar q_{+\,\beta}\,\frac{\partial^m}{\partial^+}\,. \tag{3.11}$$

They satisfy,

$$\{\,\mathcal{Q}^\alpha\,,\,q_+^\eta\,\}=-\,(\,\gamma^m\,)^{\alpha\,\eta}\partial^m\,, \tag{3.12}$$

and the supersymmetry algebra,

$$\{\,\mathcal{Q}^\alpha\,,\,\overline{\mathcal{Q}}_\eta\,\}=i\,\sqrt{2}\;\delta^\alpha{}_\eta\,\frac{1}{\partial^+}\left(\partial\,\overline{\partial}+\frac{1}{2}\,\partial^m\,\partial^m\right)\,. \tag{3.13}$$

A few central charges fit nicely into this framework. The $d = 11$ theory is known to have 517 central charges; 7 of these, may be introduced by simply replacing the seven derivatives $\partial^m$, by c-numbers $Z^m$. The remaining 510 charges have an eleven-dimensional origin.

Having constructed the free $\mathcal{N} = 1$ SuperPoincaré generators in eleven dimensions which act on the chiral superfield, we turn to building the interacting theory.

### 3.2 The generalized derivatives

The cubic interaction in the $\mathcal{N} = 8$ Lagrangian explicitly contains the transverse derivative operators $\partial$ and $\bar\partial$. To achieve covariance in eleven dimensions, we proceed to generalize these operators as we did for $\mathcal{N} = 4$ Yang-Mills. We propose the generalized derivative

$$\overline{\nabla}\;=\;\bar\partial+\frac{\sigma}{16}\,\bar d_\alpha\,(\,\gamma^m\,)^{\alpha\,\beta}\,\bar d_\beta\,\frac{\partial^m}{\partial^+}\,, \tag{3.14}$$

which naturally incorporates the coset derivatives $\partial^m$. Here $\sigma$ is a parameter, still to be determined. We use the coset generators to produce its rotated partner $\overline{\nabla}$ by,

$$\left[\overline{\nabla}\,,\,J^m\right]\;\equiv\nabla^m=-\,i\,\partial^m+\frac{i\,\sigma}{16}\,\bar d_\alpha\,(\,\gamma^m\,)^{\alpha\,\beta}\,\bar d_\beta\,\frac{\partial}{\partial^+}\,. \tag{3.15}$$

It remains to verify that the original derivative operator is reproduced by undoing this rotation; indeed we find the required closure,

$$\left[\nabla^m\,,\,\overline{J}^{\,p}\right]\;=\;\delta^{\,m\,p}\overline{\nabla}\,. \tag{3.16}$$

The new derivative ( $\overline{\nabla}$, $\nabla^m$ ), thus transforms as a 9-vector under the little group in eleven dimensions. We note that $\sigma$ is not determined by these algebraic requirements. Instead, its value will be fixed in the next section, by requiring that our generalized vertex, satisfy invariance requirements. We define the conjugate derivative $\nabla$, by requiring that

$$\nabla\,\bar\phi\equiv\overline{(\overline{\nabla}\,\phi)}\,. \tag{3.17}$$

This tells us that,

$$\nabla \equiv \partial + \frac{\sigma^*}{16}\, d^\alpha\,(\gamma^n)^{\alpha\beta}\, d^\beta\,\frac{\partial^n}{\partial^+}\,. \tag{3.18}$$

This construction is akin to that for the $\mathcal{N}=4$ Yang-Mills theory, but this time it applies to the "oxidation" of the ($\mathcal{N}=8$, $d=4$) theory to ($\mathcal{N}=1$, $d=11$) Supergravity. This points to remarkable algebraic similarities between the two theories, with possibly profound physical consequences. It remains to show that the simple replacement of the transverse derivatives $\partial, \bar{\partial}$ by $\nabla, \overline{\nabla}$ in the ($\mathcal{N}=8$, $d=4$) interacting theory yields the fully covariant Lagrangian in eleven dimensions.

### 3.3 Invariance of the action

While the covariant ($\mathcal{N}=8$, $d=4$) Supergravity Lagrangian is known to all orders in the gravitational coupling, its light-cone superspace expression has only been constructed to first order in $\kappa$ (three-point coupling). We note that the four-point gravity vertex is indeed known, in component form [14][15] but a superfield expression remains elusive. We will work with the theory to cubic order in this section and in the next section, propose a quartic interaction term for the theory. The kinetic term is trivially made SO(9)-invariant by including the seven extra transverse derivatives in the d'Alembertian.

We start from our simplified version (2.18) of the $\mathcal{N}=8$ three-point vertex. We propose that the eleven-dimensional vertex is of the same form, but with the transverse derivatives replaced by the generalized derivatives, introduced in the previous section:

$$\mathcal{V} = -2\,\kappa \int d^{11}x \int d^8\theta\, d^8\bar{\theta}\; \frac{1}{\partial^{+2}}\,\overline{\phi}\;\;\overline{\nabla}\,\phi\,\overline{\nabla}\,\phi\,, \tag{3.19}$$

together with its complex conjugate. To show SO(9) invariance, it suffices to consider the variations

$$\delta_{J^m}\phi = \frac{i}{\sqrt{2}}\,\omega_m\,\partial^+\;\theta^\alpha(\gamma^m)_{\alpha\beta}\theta^\beta\phi\,, \tag{3.20}$$

$$\delta_{J^m}\overline{\phi} = \omega_m\left\{\frac{i}{2\sqrt{2}}\partial^+\theta^\alpha(\gamma^m)_{\alpha\beta}\theta^\beta - \frac{i}{\sqrt{2}\partial^+}\partial^\alpha(\gamma^m)_{\alpha\beta}\partial^\beta + \frac{i}{2\sqrt{2}\partial^+}d^\alpha(\gamma^m)_{\alpha\beta}d^\beta\right\}\overline{\phi}\,, \tag{3.21}$$

$$\delta_{J^m}\overline{\nabla} = -\omega_m\,\nabla^m\,, \tag{3.22}$$

where $\omega_m$ are the parameters of the SO(9)/(SO(7) $\times$ SO(2)) coset transformations. The SO(2) invariance is clear from the work in $d=4$ and the SO(7) invariance is covariantly realized so if we can show the invariance under the SO(9)/(SO(7)$\times$SO(2)) transformations we have shown invariance under the full SO(9).

#### 3.3.1 The variation

We split the calculation into three parts, based on which superfield is being varied. Terms that involve $\bar{\partial}\,\bar{\partial}$ or $\partial^m\,\partial^n$, cancel trivially. The remaining terms all involve a single SO(2) derivative and a single $\partial^m$. We list the contributions from the variations below,

**Contribution** from $\int \frac{1}{\partial^{+2}} \left( \delta_J \,\overline{\phi}\right) \overline{\nabla}\, \phi\, \overline{\nabla}\, \phi$:

$$\int \frac{i\,\sigma}{2\,\sqrt{2}} \Big\{ \frac{8}{\partial^{+3}} \overline{\phi}\; \bar{\partial}\phi \partial^m \partial^+ \,\phi - \frac{4}{\partial^{+3}} \overline{\phi} \bar{\partial}\phi \partial^m \partial^+ \phi - \\ - \frac{1}{\partial^{+3}} \overline{\phi} \bar{\partial}\phi (\gamma^m)^{\beta\kappa} (\gamma^n)^{\kappa\chi} \theta^\beta \bar{d}_\chi \partial^n \partial^+ \phi - \\ - \frac{1}{\partial^{+3}} \overline{\phi} \theta^\beta \bar{\partial}\, \partial^+\, \phi\, (\gamma^m)^{\,\beta\kappa} (\gamma^n)^{\kappa\chi}\, \bar{d}_\chi\, \partial^n \phi \Big\}. \tag{3.23}$$

**Contribution** from $\int \frac{1}{\partial^{+2}}\, \overline{\phi}\, \left( \delta_J\, \overline{\nabla} \right) \phi\, \overline{\nabla}\, \phi$:

$$\int \frac{2\,i}{\partial^{+2}}\, \overline{\phi}\; \bar{\partial}\, \phi\, \partial^m \phi\,. \tag{3.24}$$

**Contribution** from $\int \frac{1}{\partial^{+2}}\, \overline{\phi}\, \overline{\nabla} \left( \delta_J\, \phi \right) \overline{\nabla}\, \phi$:

$$\int \frac{i\,\sigma}{2\,\sqrt{2}} \left\{ \frac{4}{\partial^{+2}} \overline{\phi} \bar{\partial}\, \phi\, \partial^m\, \phi \;-\; \frac{1}{\partial^{+2}}\, \overline{\phi}\, \theta^\beta\, \bar{\partial}\, \phi\, (\gamma^m)^{\,\beta\kappa} (\gamma^n)^{\,\kappa\chi}\, \bar{d}_\chi\, \partial^n\, \phi \right\}, \tag{3.25}$$

These results are further simplified, by use of the magical identity,

$$\int \frac{1}{\partial^{+3}} \overline{\phi} \bar{\partial}\, \phi\, \partial^m\, \partial^+\, \phi = \int \frac{1}{\partial^{+3}} \overline{\phi}\, \partial^m\, \phi\, \bar{\partial} \partial^+ \phi, \tag{3.26}$$

which follows from the duality constraint and numerous partial integrations. The final form of the variation then reads,

$$\delta_J \mathcal{V}\, \propto \int \left( \frac{i\,\sigma}{\sqrt{2}} + i \right) \frac{1}{\partial^{+2}}\, \overline{\phi}\, \bar{\partial}\, \phi\, \partial^m\, \phi\,, \tag{3.27}$$

Eleven-dimensional Poincaré invariance requires that this variation vanish. This determines $\sigma$ and hence the generalized derivative,

$$\overline{\nabla} = \bar{\partial} - \frac{1}{8\,\sqrt{2}}\, \bar{d}_\alpha\, (\,\gamma^m\,)^{\alpha\,\beta} \bar{d}_\beta\, \frac{\partial^m}{\partial^+}\,. \tag{3.28}$$

This completes the proof of SO(9) invariance for the three-point function. It is rather remarkable that this simple replacement of the transverse derivatives, renders the action covariant in eleven dimensions. In this light-cone form, the Lorentz invariance in eleven dimensions is automatic once the little group invariance has been established. We have therefore proven eleven-dimensional invariance to order $\kappa$.

## 4. Extension to order $\kappa^2$

The next step in this program, is to extend the oxidation procedure to order $\kappa^2$. However, the starting point for this process, the ($\mathcal{N}=8$, $d=4$) four-point interaction (in light-cone superspace) is unknown. In this section, we outline a general procedure to derive the four-dimensional quartic interaction. Once this vertex is determined, we expect the oxidation procedure to follow along very similar lines (thus yielding, the ($\mathcal{N}=1, d=11$) action to order $\kappa^2$).

### 4.1 Conjectured quartic vertex

The similarity between the three-point functions of the $\mathcal{N}=4$ Yang-Mills and $\mathcal{N}=8$ Supergravity theories is quite suggestive. Based on this comparison, we conjecture that the ($\mathcal{N}=8, d=4$) four-point vertex is simply,

$$\mathcal{V}=\kappa^2\int\left\{\,\overline{\phi}\,\phi\bar{\partial}\,\phi\,\partial\,\overline{\phi}\;+\;\beta\,\frac{1}{\partial^{+2}}\,(\,\bar{\partial}\,\phi\,\partial^{+2}\,\phi\,)\,\frac{1}{\partial^{+2}}\,(\,\partial\,\overline{\phi}\,\partial^{+2}\,\overline{\phi}\,)\right\} \tag{4.1}$$

where $\beta$ remains to be fixed. A direct check of this result can be achieved by comparison with the component form of light-cone gravity. As mentioned in the previous section, the four-point function for gravity in light-cone gauge is known [14], but there exists no expression for the quartic(or higher) vertices in terms of superfields in the literature. In component form, it is well known that the light-cone time derivative $\partial_+$, makes an appearance at every order and needs to be eliminated via field-redefinitions. This redefinition needs to be understood from a superspace point of view. We hope to return to these issues in a future publication [16]. Assuming this four-point vertex (in four dimensions), its "oxidized" eleven-dimensional version is conjectured to be,

$$\mathcal{V}=\kappa^2\int\left\{\overline{\phi}\,\phi\,\overline{\nabla}\,\phi\,\nabla\,\overline{\phi}\;+\;\beta\,\frac{1}{\partial^{+2}}\,(\,\overline{\nabla}\,\phi\,\partial^{+2}\,\phi\,)\,\frac{1}{\partial^{+2}}\,(\,\nabla\,\overline{\phi}\,\partial^{+2}\,\overline{\phi}\,)\right\}. \tag{4.2}$$

It remains to be seen if this simple form reproduces the full quartic vertex of eleven-dimensional supergravity.

### 4.2 Supersymmetry variations

A possible way to determine the four-point vertex is to start from the known two- and three-point vertices and require invariance of the action (to order $\kappa^2$) under the non-linear dynamical SuperPoincaré transformations. Simplest among these are the dynamical supersymmetry generators, already derived up to first order in the coupling constant for the ($\mathcal{N}=8, d=4$) theory in reference [13].

$$\begin{aligned}
\delta_{\bar{q}_-}{}^{0}\,\phi &= \frac{\partial}{\partial^+}\,\bar{q}_+\,\phi\\
\delta_{\bar{q}_-}{}^{\kappa}\,\phi &= -2\,\kappa\,\frac{1}{\partial^+}\left\{\,\bar{\partial}\,\frac{\partial}{\partial\,\theta}\,\phi\,\partial^{+2}\,\phi\;-\;\partial^+\,\frac{\partial}{\partial\,\theta}\,\phi\,\partial^+\,\bar{\partial}\,\phi\,\right\}.
\end{aligned} \tag{4.3}$$

This variation, clearly preserves the chirality of the superfield it acts on. Note that we can replace the $\frac{\partial}{\partial\theta}$ in the above expression by $\bar{d}$ or $\bar{q}_+$. This is possible because the additional $\bar{\theta}\,\partial^+$ piece cancels between the two terms in the variation.

To determine the four-point function, we need the dynamical supersymmetry variation at order $\kappa^2$. We do not yet know its exact form but have narrowed it down, based on the following requirements:

- Helicity $= +\frac{5}{2}$ (based on the variations at order 0 and $\kappa$)
- Dimensions $= [L]^{\frac{1}{2}}$ (again, based on the earlier variations)
- The chirality of the superfield it acts on must not be affected
- Finally, it must preserve the "inside-out" superfield constraints.

The requirement that it leave chirality invariant, is satisfied through Chiralization, a procedure we explain in section 4.3. The first three constraints offer an Ansatz for the variation,

$$\delta_{\bar{q}_-}{}^{\kappa^2}\,\phi \,\propto\, \kappa^2\,\frac{1}{\partial^{+n}}\left\{\partial^{+a}\,\bar{d}_\beta\,\phi\,\partial^{+b}\,\phi\,\partial^{+c}\,\partial\,\bar{\phi}\right\}+\cdots \tag{4.4}$$

with the restriction, $a+b+\,c=3+n$ (the $\cdots$ signify that more terms need to be added to ensure chirality). Once this expression is known, the four-point interaction is easy to determine. In a future publication [16], we will derive the exact form of $\delta_{\bar{q}_-}{}^{\kappa^2}$ and prove in addition, closure of the supersymmetry algebra with the SuperPoincaré generators.

The lowest order dynamical supersymmetry variations for the $(\mathcal{N}=1, d=11)$ theory were detailed in equation (3.11). We conjecture that at first order in coupling, the variations are obtained by simply oxidizing the four-dimensional result above. That is,

$$\delta_{\bar{\mathcal{Q}}_-}{}^{\kappa}\,\phi = -2\kappa\,\frac{1}{\partial^+}\left\{\overline{\nabla}\,\frac{\partial}{\partial\,\theta}\phi\partial^{+2}\,\phi - \partial^+\,\frac{\partial}{\partial\,\theta}\phi\partial^+\overline{\nabla}\,\phi\right\}. \tag{4.5}$$

This result needs to be checked but we believe it to be correct since it serves as a bridge between the two- and three-point vertices.

### 4.3 Chiralization

"Chiralization" is a descent procedure whereby non-chiral expressions are rendered chiral. For any general non-chiral expression of the form, $A\,\bar{B}$ (where $A$ is any compound chiral function and $\bar{B}$ a compound anti-chiral function), we define a "chiral product" through a descent relation in chiral derivatives,

$$\mathcal{C}\,(\,A\bar{B}\,) = A\,\bar{B} + \sum_{n=1}^{8}\frac{(-1)^n}{n!}\,\frac{\bar{d}_{\alpha_1\,\cdots\,\alpha_n}}{(-\,i\,\sqrt{2}\,\partial^+)^n}\,(\,A\,d^{\alpha_n\,\cdots\,\alpha_1}\,\bar{B}\,)\,, \tag{4.6}$$

where $\bar{d}_{\alpha_1\,\cdots\,\alpha_n} = \bar{d}_{\alpha_1}\,\cdots\,\bar{d}_{\alpha_n}$ and $d^{\alpha_n\,\cdots\,\alpha_1} = d^{\alpha_n}\,\cdots\,d^{\alpha_1}$.

$\mathcal{C}\,(\,A\bar{B}\,)$ is now a chiral function and satisfies, $d\,\mathcal{C} = 0$. Clearly, the descent series involves as many terms as there are supersymmetries in the theory.



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This procedure is equally applicable to other non-chiral forms. For example, the product, $\bar{d}\,A\,B$ where both $A$ and $B$ are chiral functions, is chiralized by the addition of the term,

$$-\,\partial^+\,A\,\frac{\bar{d}}{\partial^+}\,B\,. \tag{4.7}$$

Similarly, the addition of the two terms,

$$-\,\partial^+\,\bar{d}\,A\,\frac{\bar{d}}{\partial^+}\,B\;+\;\partial^{+2}\,A\,\frac{\bar{d}\,\bar{d}}{\partial^{+2}}\,B \tag{4.8}$$

to the expression $\bar{d}\,\bar{d}\,A\,B$, chiralizes it and so on.

This procedure is invaluable when dealing with variations (with respect to supersymmetry and the boosts) at higher orders. This simple recipe ensures that all variations, respect the superfield chirality structure.

## 5. Conclusions

Eleven-dimensional Supergravity has been successfully formulated in light-cone superspace to order $\kappa$. Once again, the key to this formulation is the generalized derivative whose components transform as an SO(9) vector. Further progress necessarily requires the determination of the quartic interaction. We will describe the procedure to derive these higher-point vertices in superspace, in a future publication [16].

Based on Curtright's reasoning [3], the divergence of the ($\mathcal{N}=8, d=4$) theory may be attributed to the incomplete cancelation of the Dynkin indices(starting with the eight-order invariant) of the SO(9) representations. The eight-order invariant first appears at three loops and hence the divergent nature of the theory necessarily begins at three loops or higher. Our formulation of the $\mathcal{N}=8$ theory provides an ideal framework to verify this explicitly, because the role of the little-group is made apparent on the light-cone. We hope to come back to this analysis and compare our results with existing predictions in the literature. In reference [17] for example, the authors argue that no three-loop divergence appears and further, that the first counterterm occurs at five loops.

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## A. Appendix

With the space-time metric $(-,+,+,\ldots,+)$, the light-cone coordinates and their derivatives are

$$x^{\pm}\;=\;\frac{1}{\sqrt{2}}\,(\,x^0\pm x^3\,)\,;\qquad \partial^{\pm}\;=\;\frac{1}{\sqrt{2}}\,(-\partial_0\pm\partial_3\,)\,; \tag{A.1}$$

$$x\;=\;\frac{1}{\sqrt{2}}\,(\,x_1+i\,x_2\,)\,;\qquad \bar{\partial}\;=\;\frac{1}{\sqrt{2}}\,(\,\partial_1-i\,\partial_2\,)\,; \tag{A.2}$$

$$\bar{x}\;=\;\frac{1}{\sqrt{2}}\,(\,x_1-i\,x_2\,)\,;\qquad \partial\;=\;\frac{1}{\sqrt{2}}\,(\,\partial_1+i\,\partial_2\,)\,, \tag{A.3}$$



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such that

$$\partial^+ x^- = \partial^- x^+ = -1\,; \qquad \bar\partial\, x = \partial\, \bar x \,= +1\,. \tag{A.4}$$

Anti-commuting Grassmann variables $\theta^\alpha$ and their conjugates, $\bar\theta_\alpha(\alpha = 1, 2, \ldots, 8)$ are defined by,

$$\{\,\theta^\alpha\,,\,\theta^\beta\,\} \;=\; \{\bar\theta_\alpha\,,\,\bar\theta_\beta\,\} \;=\; \{\bar\theta_\alpha\,,\,\theta^\beta\,\} \;= 0\,, \tag{A.5}$$

$\theta^\alpha$ transforms according to the real spinor representation of SO(7). Despite the reality of the spinor representation, we continue to use the “up” and “down” indices for notational convenience. Their derivatives are written as

$$\bar\partial_\alpha \;\equiv\; \frac{\partial}{\partial\,\theta^\alpha}\,; \qquad \partial^\beta \;\equiv\; \frac{\partial}{\partial\,\bar\theta_\beta}\,, \tag{A.6}$$

with canonical anticommutation relations

$$\{\,\partial^\alpha\,,\,\bar\theta_\beta\,\} \;=\; \delta^\alpha{}_\beta\,; \qquad \{\,\bar\partial_\alpha\,,\,\theta^\beta\,\} \;=\; \delta_\alpha{}^\beta\,. \tag{A.7}$$

Under conjugation, upper and lower spinor indices are interchanged, so that $\overline{\theta^\alpha} = \bar\theta_\alpha$, while

$$\overline{(\bar\partial_\alpha)} = -\partial^\alpha\,; \qquad \overline{(\partial^\beta)} = -\bar\partial_\beta\,. \tag{A.8}$$

Also, the order of the operators is interchanged; that is $\overline{\theta^\alpha\theta^\beta} = \bar\theta_\beta\,\bar\theta_\alpha$, and $\overline{\partial^\alpha\,\partial^\beta} = \bar\partial_\beta\,\bar\partial_\alpha$. The chiral derivatives in superspace are,

$$d^{\,\alpha} = -\partial^\alpha - \frac{i}{\sqrt 2}\,\theta^\alpha\,\partial^+\;; \qquad \bar d_{\,\beta} = \;\; \bar\partial_\beta + \frac{i}{\sqrt 2}\,\bar\theta_\beta\partial^+\,, \tag{A.9}$$

and satisfy,

$$\{\,d^\alpha\,,\,\bar d_\beta\,\} = -i\,\sqrt 2\,\delta^\alpha{}_\beta\,\partial^+\,. \tag{A.10}$$

The SO(7) $\gamma$-matrices are defined by

$$\mathrm{Tr}(\gamma^m\gamma^n) = 8\delta^{mn}\,. \tag{A.11}$$

We also define,

$$\gamma^{mn} = \frac{1}{2}[\gamma^m\,,\,\gamma^n\,]\,, \qquad m \neq n\,. \tag{A.12}$$

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